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PULSE COMPRESSION NATURE IN A STRONGLY NONLINEAR GRAINED MEDIUM

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A new class of structurized media with zero long-wave velocity of sound is considered, for which the classical wave equation is unacceptable. A nonlinear term is rather a basic one than plays the role of a small correction as with Korteveg-de-Vries equation. The nature of collective excitations in such media has been studied.

Key words: Nonlinearity, Grained medium, Solitons.

The disturbance propagation through discrete strongly nonlinear media, where nonlinearity is the most contributive factor and does not serve as a small correction to a linear description, was studied both theoretically and experimentally in [1-5]. Their characteristic feature is that long-wave velocity of sound c_0 is equal to zero, initial deformation being also zero. The classical long-wave equation

$$u_{tt} = c_0^2 u_{xx} \tag{1}$$

where u is the displacement, c_0 is the long-wave velocity of sound, is unsuitable for the description of wave motions because $c_0 = 0$. A nonlinear wave equation

$$u_{tt} = c_0^2 u_{xx} + 2c_0\gamma u_{xxxx} - \varepsilon u_x u_{xx}, \tag{2}$$

which is obtained by substituting small nonlinear and dispersive terms in (1), is also unacceptable in these cases, because γ and ε dependent on c_0 also vanish [1]. Thus, in the study of compression pulses nature in strongly nonlinear discrete media there arises the problem of continual decription of them without the use of (1) and (2) as basic equations.

For ordered one-dimensional system of grains with radii $R = a/2$ interacting by the Hertz law, the following equation has been obtained instead of (2) in the long-wave approximation for compression wave [1-2]:

$$u_{tt} = c^2\left\{\frac{3}{2}(-u_x)^{1/2}u_{xx} + \frac{a^2}{8}(-u_x)^{1/2}u_{xxxx} - \frac{a^2}{8}\frac{u_{xx}u_{xxx}}{(-u_x)^{1/2}} - \frac{a^2}{64}\frac{(u_{xx})^3}{(-u_x)^{3/2}}\right\}, \tag{3}$$

$$-u_x > 0, \quad c^2 = \frac{2E}{\pi\rho_0(1 - \nu^2)}$$

(here ρ_0 is the density, E, ν are the Young's modulus and Poisson's coefficient, respectively).

The main results obtained in [1-2] for steady-state solutions of (3) are as follows.

Equation (3) has an exact solution in the form of a periodic wave for deformations $\xi = -u_x$,

$$\xi = \left[\frac{5}{4} \frac{V^2}{c^2} \right]^2 \cos^4 \frac{\sqrt{10}}{5a} x$$

with the characteristic space period

$$L \approx \frac{5a}{\sqrt{10}} \pi \approx 5a.$$

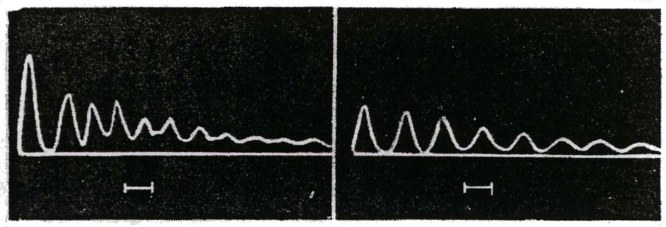
If an initial deformation in a matter is $\xi_0 > 0$, solitons of a new type may develop, which qualitatively differ from the KdV solitons being steady-state solutions of (2) when its time derivative order decreases. The characteristic space size of solitons at a maximum deformation $\xi_m \gg \xi_0$ is equal to L, and the correlation between ξ_m and phase velocity is expressed as follows:

$$\xi_m = \left(\frac{5}{4} \frac{V^2}{c^2} \right)^2.$$

As shown by the numerical calculations of a discrete chain, the solitary waves (steady-state solutions of (3)) are really of a soliton-like character. The problem of a piston, decay of different initial disturbances, nature of compression pulses with randomly varying particle size, as well as a qualitative effect of a preliminary static loading of this system on the pulse character were studied in [1-5].

A long-wave equation for a periodical system with two different masses m_1 and m_2 is analogous to (3) except for the coefficients dependent on the m_1/m_2 ratio. When $m_1 \gg m_2$ the soliton-like solutions for this case are also analogous to that for (3).

Both analytical and numerical soliton-like solutions of this strongly nonlinear system were also confirmed by experiment [1,3-5].



a b

This Figure illustrates the variations in pressure acting on a rigid wall bounding the chain consisting of 40 (a) and 80 (b) steel particles 4.75 mm in dia. under the impact of a steel piston with a velocity of 1 m/s upon a free end of the wall, the piston mass being 10 times as high as that of the particle. The time scale is 25 mcs/div. As is seen, the amplitude decays, that was not taken into account in the calculations.

It is interesting to clear up what behaviour is characteristic for the waves in such media in the general case when the coefficient in the low of interaction between neighbouring particles is $n \neq 3/2$. The simplest one-dimensional structures are thereby used as an example.

The equation of motion of the i-th particle is of the form

$$\ddot{u}_i = A\left(u_{i-1} - u_i\right)^n - A\left(u_i - u_{i+1}\right)^n,$$

$$N \geqslant i \geqslant 2, \qquad (4)$$

where u_i is the particle displacement from an equilibrium state, A is constant of interaction, N the number of particles. Assuming that $L \gg a$ (a is the distance between the centres of the particles in the nondeformed system), one may obtain from (4) [2] for $n > 0$ a continual nonlinear long-wave equation, in which the terms of the order $(a/L)^2$ relative to the basic one have been remained and the convective derivative of a velocity has been omitted [6]:

$$u_{tt} = \qquad (5)$$

$$= c_n^2 n\Big\{(-u_x)^{n-1}u_{xx} + \frac{a^2}{12}(-u_x)^{n-1}u_{xxxx} - \frac{a^2}{6}(n-1)(-u_x)^{n-2}u_{xx}u_{xxx} + \frac{a^2}{24}(n-1)(n-2)(-u_x)^{n-3}u_{xx}^3\Big\},$$

$$c_n^2 = Aa^{n+1}, \quad -u_x > 0, \; n > 0.$$

Ignoring the convective derivative is wittingly valid when $V \ll c_n$.

For steady-state solutions of the form $u(x-Vt)$, after having introduced the term of deformation $\xi = -u_x$ from (5), we obtain

$$\frac{V^2}{c_n^2 n}\xi_x = . \qquad (6)$$

$$= \xi^{n-1}\xi_x + \frac{a^2}{6(n+1)}\left\{\xi^{\frac{n-1}{2}}\left[\xi^{\frac{n+1}{2}}\right]_{xx}\right\}_x.$$

Integration of (6) over x and substitution of variables $(\xi,x) \to (y,\eta)$ give the equation analogous to that of oscillation motion in a potential field $W(y)$:

$$y_{\eta\eta} = -\frac{\partial}{\partial y}W(y), \qquad (7)$$

$$W(y) = \frac{y^2}{2} - \frac{(n+1)}{4}y^{\frac{4}{n+1}} + C\,y^{\frac{2}{n+1}},$$

$$\eta = \frac{x}{a}\sqrt{\frac{6(n+1)}{n}}, \quad y = \xi^{\frac{n+1}{2}}\left[\frac{c_n}{V}\right]^{\frac{n+1}{n-1}},$$

where C is constant.

Equation (7) when $C = 0$ has a steady-state solution in the form of a periodic wave when $n > 1$:

$$\xi = \left\{\frac{(n+1)}{2}\frac{V^2}{c_n^2}\right\}^{\frac{1}{n-1}} \times$$

$$\times \sin^{\frac{2}{n-1}}\left[\frac{(n-1)}{(n+1)}\sqrt{\frac{6(n+1)}{n}}\,\frac{x}{a}\right].$$

If $0 < C < (n^2-1)n^{\frac{n}{1-n}}/2 = k$, equation (7) has soliton-like solutions. When $C \lesssim k$, the latter coincides with the KdV solitons, and when $C \to 0$ ($\xi_m \gg \xi_0 \neq 0$) they are qualitatively different as compared to the latter ones. In this case the ξ_m dependence and the characteristic size L_n are of the form

$$\xi_m = \left\{\frac{(n+1)}{2}\frac{V^2}{c_n^2}\right\}^{\frac{1}{n-1}}, \quad L_n = \frac{\pi a}{n-1}\sqrt{\frac{n(n+1)}{6}},$$

$$n > 1. \qquad (8)$$

When $n = 1 + \psi$ ($0 < \psi \ll 1$) it is evident that solitons (8) are qualitatively different from linear nearly-sonic disturbances characteristic for

$n = 1$ and the KdV solitons. Stationary solitary compression waves of the type (8) are the excitation background of intensive pulses of a given discrete strongly nonlinear system and are to be especially called the nestones [1,6]. For them $\xi_m \gg \xi_0$, when $\xi_m \to \xi_0$ the nestones are transformed into the KdV solitons, and when $\xi_m \approx \xi_0$ they are transformed into sonic disturbances.

The existence of such steady-state solutions for a continuous spectrum of values $n > 1$ stimulates the development of the research in a qualitatively new field of strongly nonlinear wave dynamics. The problem of correctness of introducing the notion of a sonic wave for real media should be particularly emphasized, since the latter is based on an exact equation $n = 1$.

When $n = 1 + \phi$ $(0 < \phi \ll 1)$, a steady-state solution is represented by the nestone (8) with the "quantized" spatial size $L_{1\psi}$ and quantity ξ_m rather than by sonic wave:

$$L_{1\psi} \approx \frac{\pi a}{\sqrt{3}\,\phi}, \quad \xi_m = \left(\frac{V^2}{c_1^2}\right)^{1/\psi}$$

CONCLUSION

A qualitatively new class of solitary waves, nestones, being steady-state solutions of equation (5), has been shown to exist for strongly nonlinear disturbances of discrete media, where nonlinearity is the most contributive factor. These waves are the excitation background for "sonic vacuum" when $\xi_m \gg \xi_0$ $(V \gg c_0)$. It is natural that they appear due to the increase in the amplitude of the impact on any discrete strongly nonlinear system. It is most probable that such disturbances may develop in the media whose compressibility significantly increases with pressure, e.g., in molecular crystals. The stationary waves described in this paper are qualitatively different from the previously known ones, therefore, new approaches to the description of a strongly nonlinear wave dynamics need to be developed.

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